Information Geometry and quantum mechanics

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The space $\mathcal{P}_n^\times$ of non vanishing probabilities $p : E_n \to \mathbb{R}$ defined on a finite set $E_n := \{x_1, ..., x_n\}$, i.e.,

$$\mathcal{P}_n^\times := \Big\{ p : E_n \to \mathbb{R} \,\big|\, p > 0 \text{ and } \sum_{k=1}^{n} p(x_k) = 1 \Big\}, \tag{1}$$

is a very well known statistical manifold; it is endowed with the so-called *Fisher metric* g_F and *exponential connection* $\nabla^{(e)}$, and together, $(\mathcal{P}_n^\times, g_F, \nabla^{(e)})$ form a example of *dually flat structure*.

In this talk, we will use the dually flat structure of $\mathcal{P}_n^\times$ to construct a Kähler structure on $T\mathcal{P}_n^\times$ via Dombrowski's construction, and we will explain how a significant part of the quantum mechanical formalism (for the finite dimensional Hilbert space $\mathcal{H} = \mathbb{C}^n$) can be described using the Kähler geometry of $T\mathcal{P}_n^\times$.

If time allows, we should also consider an infinite dimensional example.