Possible goals for Projects

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In my lectures I will cover the construction of **Minimal Surfaces** via the Weiersraß representation. This depends heavily on complex analysis.

Complex functions are visualized as conformal maps. I show with pictures from 3DXplorMath that a better choice of a mapping grid will reveal more properties of the complex function. The inverse of complex functions quickly leads to multivalued maps, like $z \mapsto \sqrt{z}$ or $z \mapsto \log z$. To deal with these needs the concept of **analytic continuation**.
A simple task is: Write programs for analytic continuation of these two functions and design demo programs which show how they work.
Minimal surfaces like the Trinoid need parametrizations of the sphere which have three polar coordinate centers. Program such grids with the help of the complex square root.
The Weierstraß representation maps a piece of the complex plane (or of some more complicated Riemann surface) to a minimal surface in $\mathbb{R}^3$. The simplest minimal surfaces have the complex plane as domain and their Weierstraß data are polynomial. One can visualize them by integrating the data over a polar coordinate grid.
A polar coordinate grid is also suitable for surfaces which are defined on $\mathbb{C} \setminus \{0\}$. The programming problems are similar to the previous ones.
More interesting minimal surfaces like the Trinoid still have rational Weierstraß data, but require more complicated grids for the integration. For example polar centers at the third roots of unity for the Trinoid.
Since standard integration routines do not accept multivalued functions one needs to write a program which does the analytic continuation together with the integration. For example integrate the square root of a polynomial along curves which avoid the zeros of the polynomial.
One can use Matlab routines to show the computed surfaces.
For more interesting minimal surfaces one has to understand tori as Riemann surfaces. At first one needs only slight modifications of standard grids: small detours around the points where the chosen coordinate function has derivative 0, i.e. fails to be a coordinate. The Weierstraß integration however has to handle multivalued functions.

There are also geometrically interesting programming tasks which are not related to minimal surfaces.
A program which does parallel translation in the normal bundle of space curves immediately gives interesting tubes and animations. If one translates conformal grids which are given in one normal plane then one gets entertaining examples of triply orthogonal systems of surfaces.
Red-green stereo projections improve the 3-dimensional intuition. Two easy beginnings are:
Stereo cloud representations of simple parametrized surfaces.
Sequences of pictures which explain the Platonic solids.
Surfaces of higher genus have no 'good' parametrizations and even poor parametrizations are difficult to find. Therefore one has with the stereo cloud visualizations an easily applicable tool. – The method works because of the following observation: If one intersects three pairwise orthogonal random families of uniformly distributed lines with the surface then one obtains random points on the surface whose density varies by less than a factor 2.