Given a holomorphic map-germ $f : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}, 0)$ with a critical value at $0 \in \mathbb{C}$, there are two equivalent ways of defining its Milnor fibration. The first is:

$$\phi = \frac{f}{|f|} : S_{\varepsilon} \setminus K \rightarrow S^1,$$

where $K = f^{-1}(0) \cap S_{\varepsilon}$ is the link. The other, given essentially by Milnor himself by:

$$f : N(\varepsilon, \eta) \rightarrow \partial \mathbb{D}_{\eta},$$

where $\varepsilon >> \eta > 0$ are sufficiently small, $\mathbb{D}_{\eta} \subset \mathbb{C}$ is the disc of radius $\eta$ around $0 \in \mathbb{C}$, $\mathbb{B}_{\varepsilon}$ is the ball of radius $\varepsilon$ around $0 \in \mathbb{C}$, and $N(\varepsilon, \eta)$ is the Milnor tube $\mathbb{B}_{\varepsilon} \cap f^{-1}(\partial \mathbb{D}_{\eta})$. (We remark that Milnor only proved that the fibres of (2) are equivalent to those of (1), and not that (2) is actually a fibre bundle; this was certainly known to Milnor when $f$ has an isolated critical point, and later completed by Lê.)

We show that there is a canonical decomposition of the whole ball $\mathbb{B}_{\varepsilon}$ into real analytic hypersurfaces $X_\theta$ that spin around their “axis” $V_{\varepsilon} = f^{-1}(0) \cap \mathbb{B}_{\varepsilon}$ forming a kind of “open-book” with singular binding. Using this canonical decomposition we give a refinement of Milnor Fibration Theorem.

Joint work with J. Seade and J. Snoussi.